

Verifying differential equations satisfied by Jacobi elliptic functions am , sn , cn , dn for $k > .1$ Assuming $m = k^2$, $k > 1$ From the definition:

$$\text{sn}(u, m) = \frac{1}{k} \text{sn}(ku, 1/m),$$

with $1/m < 1$. We also know that

$$\text{cn}(u, m) = \text{dn}(ku, 1/m),$$

$$\text{dn}(u, m) = \text{cn}(ku, 1/m).$$

Thus

$$\text{sn}'(u, m) = \frac{1}{k} k \text{sn}'(ku, 1/m) = \text{cn}(ku, 1/m) \text{dn}(ku, 1/m) = \text{dn}(u, m) \text{cn}(u, m).$$

$$\begin{aligned} \text{cn}'(u, m) &= k \text{dn}'(ku, 1/m) = k(-(1/m) \text{sn}(ku, 1/m) \text{cn}(ku, 1/m)) \\ &= -(1/k) \text{sn}(u, 1/m) \text{cn}(ku, 1/m) = -\text{sn}(u, m) \text{cn}(u, m), \end{aligned}$$

$$\begin{aligned} \text{dn}'(u, m) &= k \text{cn}'(ku, 1/m) = -k \text{sn}(ku, 1/m) \text{dn}(ku, 1/m) \\ &= -(1/k) \text{sn}(u, 1/m) \text{cn}(ku, 1/m) = -\text{sn}(u, m) \text{dn}(u, m). \end{aligned}$$

Recall that for $m > 1$ am was defined as

$$\text{am}(u, m) = \arcsin \text{sn}(u, m).$$

Therefore

$$\text{am}'(u, m) = \frac{1}{\sqrt{1 - \text{sn}^2(u, m)}} \text{sn}'(u, m) = \frac{1}{\text{cn}(u, m)} \text{cn}(u, m) \text{dn}(u, m) = \text{dn}(u, m)$$

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